

10 COMPARING ALTERNATIVE SYSTEMS

The great tragedy of Science—the slaying of a beautiful hypothesis by an ugly fact.

—Thomas H. Huxley

In this lab we see how ProModel is used with some of the statistical methods presented in Chapter 10 to compare alternative designs of a system with the goal of identifying the superior system relative to some performance measure. We will also learn how to program ProModel to use common random numbers (CRN) to drive the simulation models that represent alternative designs for the systems being compared. The use of CRN allows us to run the opposing simulations under identical experimental conditions to facilitate an objective evaluation of the systems.

L10.1 Overview of Statistical Methods

Often we want to compare alternative designs of a system. For example, we may wish to compare two different material handling systems, production scheduling rules, plant layouts, or staffing schedules to determine which yields better results. We compare the two alternative system designs based on a certain measure of performance—for example, production rate, number of customers served, time in system, or waiting time for service. The comparison is based on the estimates of the expected value of the performance measure that we derive from simulating the alternative system designs. If the estimates provide evidence that the performances of the two alternative simulation models are not equal, we can justify picking the alternative that produced the better estimate as the best alternative.

Chapter 10 covers the paired-*t* confidence interval method and the Welch confidence interval method for testing hypotheses about two alternatives. The

Bonferroni approach is used for comparing from three to about five alternatives. Either the Welch method or paired- t method is used with the Bonferroni approach to make the comparisons. The Bonferroni approach does not identify the best alternative but rather identifies which pairs of alternatives perform differently. This information is helpful for identifying the better alternatives, if in fact there is a significant difference in the performance of the competing alternatives.

Chapter 10 recommends that the technique of analysis of variance (ANOVA) be used when more than about five alternative system designs are being compared. The first step in ANOVA is to evaluate the hypothesis that the mean performances of the systems are equal against the alternative hypothesis that at least one pair of the means is different. To figure out which means differ from which other ones, a multiple comparison test is used. There are several multiple comparison tests available, and Chapter 10 presents the Fisher's protected least significant difference (LSD) test.

The decision to use paired- t confidence intervals, Welch confidence intervals, the Bonferroni approach, or ANOVA with protected LSD depends on the statistical requirements placed on the data (observations from the simulations) by the procedure. In fact, simulation observations produced using common random numbers (CRN) cannot be used with the Welch confidence interval method or the technique of ANOVA. Please see Chapter 10 for details.

In this lab we will use the paired- t confidence interval method because it places the fewest statistical requirements on the observations collected from the simulation model. Therefore, we are less likely to get into trouble with faulty assumptions about our data (observations) when making decision based on paired- t confidence intervals. Furthermore, one of the main purposes of this lab is to demonstrate how the CRN technique is implemented using ProModel. Out of the comparison procedures of Chapter 10, only the ones based on paired- t confidence intervals can be used in conjunction with the CRN technique.

L10.2 Three Alternative Systems

Problem Statement

The analysis of the Spuds-n-More by the Seashore restaurant started in Section L9.2 of Lab Chapter 9 will be completed in this lab. The details of the system and objective for the study are reviewed here. Spuds-n-More, a takeout fast-food restaurant, is a favorite among vacationers at a seaside resort. Customers walking along the boardwalk access the restaurant via a single ordering window, where they place their orders, pay, and receive their food. There is no seating in the restaurant. The interarrival time of customers to Spuds-n-More is exponentially distributed with a mean of six minutes. The activity times are given in Table L10.1. The waiting area in front of the walkup window can accommodate only five customers. If the waiting area is filled to capacity when a customer arrives, the customer bypasses Spuds-n-More. Mr. Taylor closes the entry to the waiting area in front of his restaurant after eight hours of operation. However, any customer

already in the waiting area by closing time are served before the kitchen shuts down for the day.

Mr. Taylor is thinking about expanding into the small newspaper stand next to Spuds-n-More, which has been abandoned. This would allow him to add a second window to serve his customers. The first window would be used for taking orders and collecting payments, and the second window would be used for filling orders. Before investing in the new space, however, Mr. Taylor needs to convince the major stockholder of Spuds-n-More, Pritchard Enterprises, that the investment would allow the restaurant to serve more customers per day.

A baseline (as-is) model of the current restaurant configuration was developed and validated in Section L9.2 of Lab Chapter 9. See Figures L9.1 and L9.2 for the layout of the baseline model and the printout of the ProModel model. Our task is to build a model of the proposed restaurant with a second window to determine if the proposed design would serve more customers per day than does the current restaurant. Let's call the baseline model Spuds-n-More1 and the proposed model Spuds-n-More2. The model is included on the CD accompanying the book under file name Lab 10_2 Spuds-n-More2.MOD.

The ProModel simulation layout of Spuds-n-More2 is shown in Figure L10.1, and the ProModel printout is given in Figure L10.2. After customers wait in the order queue (location Order_Q) for their turn to order, they move to the first window to place their orders and pay the order clerk (location Order_Clerk). Next customers proceed to the pickup queue (location Pickup_Q) to wait for their turn to be served by the pickup clerk (Pickup_Clerk) at the second window. There

FIGURE L10.1
Layout of Spuds-n-More2 simulation model.

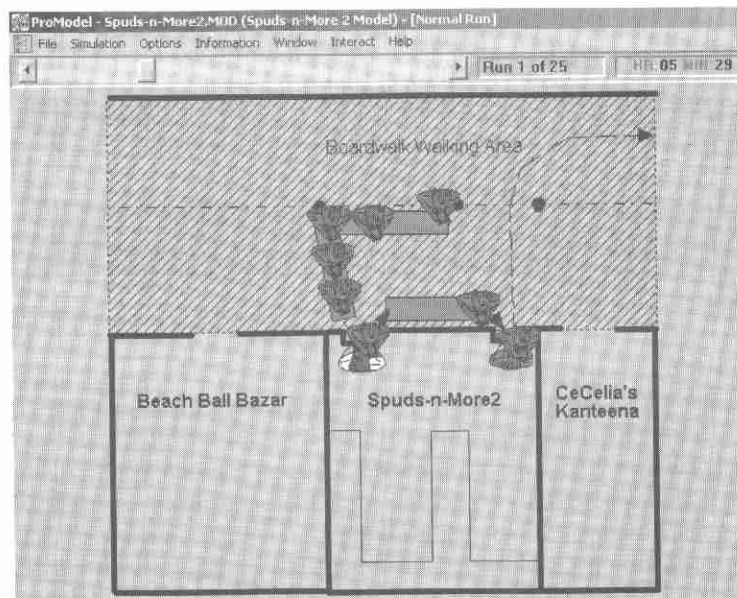


FIGURE L10.2

The Spuds-n-More2 simulation model.

```

Time Units: Minutes
Distance Units: Feet
*****
*                               Locations                               *
*****
Name      Cap Units Stats      Rules      Cost
-----
Entry     1  1  None      Oldest, ,
Order_Q   4  1  None      Oldest, FIFO,
Order_Clerk 1  1  None      Oldest, ,
Pickup_Q  1  1  None      Oldest, FIFO,
Pickup_Clerk 1  1  None      Oldest, ,
*****
*                               Entities                               *
*****
Name      Speed (fpm) Stats      Cost
-----
Customer  150      None
*****
*                               Processing                             *
*****
Process                                     Routing
-----
Entity Location Operation      Blk Output Destination Rule      Move Logic
-----
Customer Entry                1 Customer Order_Q      IF clock(hr) <= 8,1
                                Customer EXIT          IF clock(hr) > 8      If Processed = Arrivals
                                                                Then Stop

Customer Order_Q      Inc Arrivals      1 Customer Order_Clerk FIRST 1
Customer Order_Clerk // Order
                    Wait e(2) min
                    // Pay
                    Wait e(1.5) min 1 Customer Pickup_Q      FIRST 1
Customer Pickup_Q     1 Customer Pickup_Clerk FIRST 1
Customer Pickup_Clerk // Pickup
                    Wait e(5) min
                    Inc Processed 1 Customer EXIT          FIRST 1
*****
*                               Arrivals                               *
*****
Entity Location Qty Each First Time Occurrences Frequency Logic
-----
Customer Entry 1 0 inf E(6) min
*****
*                               Variables (global)                       *
*****
ID      Type      Initial value Stats
-----
Arrivals Integer 0 None
Processed Integer 0 Basic

```

is enough space for one customer in the pickup queue. The pickup clerk processes one order at a time.

As the Spuds-n-More2 model was being built, Mr. Taylor determined that for additional expense he could have a carpenter space the two customer service windows far enough apart to accommodate up to three customers in the order pickup queue. Therefore, he requested that a third alternative design with an order pickup queue capacity of three be simulated. To do this, we only need to change the

capacity of the pickup queue from one to three in our Spuds-n-More2 model. We shall call this model Spuds-n-More3. Note that for our Spuds-n-More2 and Spuds-n-More3 models, we assigned a length of 25 feet to the Order_Q, a length of 12 feet to the Pickup_Q, and a Customer entity travel speed of 150 feet per minute. These values affect the entity's travel time in the queues (time for a customer to walk to the end of the queues) and are required to match the results presented in this lab.

L10.3 Common Random Numbers

Let's conduct the analysis for Mr. Taylor using Common Random Numbers (CRN). The CRN technique provides a way of comparing alternative system designs under more equal experimental conditions. This is helpful in ensuring that the observed differences in the performance of system designs are due to the differences in the designs and not to differences in experimental conditions. The goal is to evaluate each system under the exact same set of circumstances to ensure that a fair comparison is made. Therefore, the simulation of each alternative system design is driven by the same stream of random numbers to ensure that the differences in key performance measures are due only to differences in the alternative designs and not because some designs were simulated with a stream of random numbers that produced more extreme conditions. Thus we need to use the exact same random number from the random number stream for the exact same purpose in each simulated system. A common practice that helps to keep random numbers synchronized across systems is to assign a different random number stream to each stochastic element in the model. For this reason, ProModel provides 100 unique streams (Stream 1 to Stream 100) of random numbers for generating random variates from the statistical distributions available in the software.

The Spuds-n-More models use observations (random variates) from an exponential distribution for the customer interarrival time, order time, order payment time, and order pickup time. For CRN, in this example we can use Stream 1 to generate interarrival times, Stream 2 to generate order times, Stream 3 to generate payment times, and Stream 4 to generate pickup times. The ProModel function for generating random variates from the exponential distribution is $E(a, stream)$, where the parameter a is the mean of the random variable and the parameter $stream$ is the ProModel stream number between 1 and 100. All ProModel distribution functions include the $stream$ parameter as the last parameter of the function. The $stream$ parameter is optional (you do not have to specify a value). If you do not provide a value for the $stream$ parameter, ProModel uses $stream = 1$ to generate observations (random variates) from the specified distribution. Figure L10.3 illustrates the desired assignment of random number streams in the processing and arrivals section of a Spuds-n-More model. Configure all three of your Spuds-n-More models accordingly.

FIGURE L10.3

Unique random number streams assigned to the Spuds-n-More model.

```

*****
*                               Processing                               *
*****
      Process                               Routing
-----
Entity  Location  Operation  Blk Output  Destination  Rule  Move Logic
-----
Customer Entry
      1  Customer Order_Q  IF clock(hr) <= 8,1
      Customer EXIT      IF clock(hr) > 8  If Processed = Arrivals
                               Then Stop

Customer Order_Q  Inc Arrivals  1  Customer Order_Clerk  FIRST 1
Customer Order_Clerk
// Order
Wait e(2,2) min
// Pay
Wait e(1.5,3) min 1  Customer Pickup_Q  FIRST 1
Customer Pickup_Q  1  Customer Pickup_Clerk  FIRST 1
Customer Pickup_Clerk
// Pickup
Wait e(5,4) min
Inc Processed  1  Customer EXIT  FIRST 1
*****
*                               Arrivals                               *
*****
Entity  Location  Qty  Each  First Time Occurrences  Frequency  Logic
-----
Customer Entry  1  0  inf  E(6,1) min
    
```

L10.4 Bonferroni Approach with Paired-t Confidence Intervals

Our objective is to compare the performance of the three alternative Spuds-n-More restaurant designs with respect to the number of customers processed per day. Let μ_i represent the true number of customers processed per day by restaurant design i , where $i = 1$ denotes Spuds-n-More1, $i = 2$ denotes Spuds-n-More2, and $i = 3$ denotes Spuds-n-More3. Specifically, we wish to evaluate the hypotheses

$$H_0: \mu_1 = \mu_2 = \mu_3$$

$$H_1: \mu_1 \neq \mu_2 \text{ or } \mu_1 \neq \mu_3 \text{ or } \mu_2 \neq \mu_3$$

at the $\alpha = 0.06$ significance level. To evaluate these hypotheses, we estimate the number of customers processed by each restaurant design by simulating each design for 25 days (25 replications). The simulations are to be driven by Common Random Numbers (CRN). Therefore, select the CRN option in the bottom right corner of the Simulation Options window before running the simulations (Figure L10.4). This option helps to synchronize the use of random numbers across the three simulated systems. When the option is selected, ProModel starts each replication at predetermined locations on the random number streams. At the end of a replication, ProModel advances to the next set of predetermined locations on the streams before starting the next replication. Even if one model uses more random numbers than another during a replication, both models will begin the next replication from the same positions on the random number streams. The numbers of customers processed by each restaurant design for 25 replications using the CRN option are shown in Table L10.1.

FIGUR
ProMo.
Option.
25 repl
Comm
Numbe.

TABLE

(A)

Rep.
(j)

- 1
- 2
- 3
- 4
- 5
- 6
- 7
- 8
- 9
- 10
- 11
- 12
- 13
- 14
- 15
- 16
- 17
- 18
- 19
- 20
- 21
- 22
- 23
- 24
- 25

Sample m
Sample st

FIGURE L10.4

ProModel's Simulation Options set to run 25 replications using Common Random Numbers.

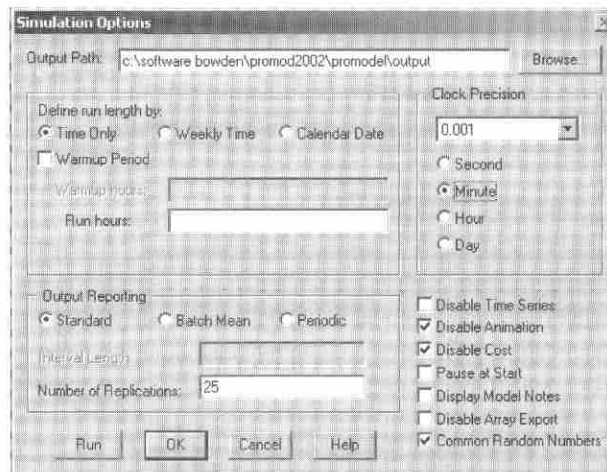


TABLE L10.1 Comparison of the Three Restaurant Designs Based on Paired Differences

(A)	(B)	(C)	(D)	(E)	(F)	(G)
Rep.	Spuds-n-More1 Customers Processed	Spuds-n-More2 Customers Processed	Spuds-n-More3 Customers Processed	Difference (B - C)	Difference (B - D)	Difference (C - D)
(i)	x_{1j}	x_{2j}	x_{3j}	$x_{(1-2)j}$	$x_{(1-3)j}$	$x_{(2-3)j}$
1	60	72	75	-12	-15	-3
2	57	79	81	-22	-24	-2
3	58	84	88	-26	-30	-4
4	53	69	72	-16	-19	-3
5	54	67	69	-13	-15	-2
6	56	72	74	-16	-18	-2
7	57	70	71	-13	-14	-1
8	55	65	65	-10	-10	0
9	61	84	84	-23	-23	0
10	60	76	76	-16	-16	0
11	56	66	70	-10	-14	-4
12	66	87	90	-21	-24	-3
13	64	77	79	-13	-15	-2
14	58	66	66	-8	-8	0
15	65	85	89	-20	-24	-4
16	56	78	83	-22	-27	-5
17	60	72	72	-12	-12	0
18	55	75	77	-20	-22	-2
19	57	78	78	-21	-21	0
20	50	69	70	-19	-20	-1
21	59	74	77	-15	-18	-3
22	58	73	76	-15	-18	-3
23	58	71	76	-13	-18	-5
24	56	70	72	-14	-16	-2
25	53	68	69	-15	-16	-1
Sample mean $\bar{x}_{(i-i')}$, for all i and i' between 1 and 3, with $i < i'$				-16.20	-18.28	-2.08
Sample standard dev $s_{(i-i')}$, for all i and i' between 1 and 3, with $i < i'$				4.66	5.23	1.61

The Bonferroni approach with paired- t confidence intervals is used to evaluate our hypotheses for the three alternative designs for the restaurant. The evaluation of the three restaurant designs requires that three pairwise comparisons be made:

- Spuds-n-More1 vs. Spuds-n-More2
- Spuds-n-More1 vs. Spuds-n-More3
- Spuds-n-More2 vs. Spuds-n-More3

Following the procedure described in Section 10.4.1 of Chapter 10 for the Bonferroni approach, we begin constructing the required three paired- t confidence intervals by letting $\alpha_1 = \alpha_2 = \alpha_3 = \alpha/3 = 0.06/3 = 0.02$.

The computation of the three paired- t confidence intervals follows:

Spuds-n-More1 vs. Spuds-n-More2 ($\mu_{(1-2)}$)

$$\alpha_1 = 0.02$$

$$t_{n-1, \alpha_1/2} = t_{24, 0.01} = 2.485 \text{ from Appendix B}$$

$$hw = \frac{(t_{24, 0.01})s_{(1-2)}}{\sqrt{n}} = \frac{(2.485)4.66}{\sqrt{25}} = 2.32 \text{ customers}$$

The approximate 98 percent confidence interval is

$$\begin{aligned} \bar{x}_{(1-2)} - hw &\leq \mu_{(1-2)} \leq \bar{x}_{(1-2)} + hw \\ -16.20 - 2.32 &\leq \mu_{(1-2)} \leq -16.20 + 2.32 \\ -18.52 &\leq \mu_{(1-2)} \leq -13.88 \end{aligned}$$

Spuds-n-More1 vs. Spuds-n-More3 ($\mu_{(1-3)}$)

$$\alpha_2 = 0.02$$

$$t_{n-1, \alpha_2/2} = t_{24, 0.01} = 2.485 \text{ from Appendix B}$$

$$hw = \frac{(t_{24, 0.01})s_{(1-3)}}{\sqrt{n}} = \frac{(2.485)5.23}{\sqrt{25}} = 2.60 \text{ customers}$$

The approximate 98 percent confidence interval is

$$\begin{aligned} \bar{x}_{(1-3)} - hw &\leq \mu_{(1-3)} \leq \bar{x}_{(1-3)} + hw \\ -18.28 - 2.60 &\leq \mu_{(1-3)} \leq -18.28 + 2.60 \\ -20.88 &\leq \mu_{(1-3)} \leq -15.68 \end{aligned}$$

Spuds-n-More2 vs. Spuds-n-More3 ($\mu_{(2-3)}$)

The approximate 98 percent confidence interval is

$$-2.88 \leq \mu_{(2-3)} \leq -1.28$$

Given that the confidence interval for $\mu_{(1-2)}$ excludes zero, we conclude that there is a significant difference in the mean number of customers processed by the

current "as-is" restaurant Spuds-n-More1 and the proposed Spuds-n-More2 design. The confidence interval further suggests that Spuds-n-More2 processes an estimated 13.88 to 18.52 more customers per day on average than does Spuds-n-More1. Likewise, there is a significant difference in the mean number of customers processed by Spuds-n-More1 and Spuds-n-More3. The proposed Spuds-n-More3 processes an estimated 15.68 to 20.88 more customers per day on average than does Spuds-n-More1. The third confidence interval suggests that the difference between Spuds-n-More2 and Spuds-n-More3, although statistically significant, is fairly small with Spuds-n-More3 processing an estimated 1.28 to 2.88 more customers per day on average than does Spuds-n-More2.

With $\alpha = 0.06$ the *overall confidence* for our conclusions is approximately 94 percent. Based on these results we are justified in concluding that both of the new designs (Spuds-n-More2 and Spuds-n-More3) serve more customers per day than does the current "as-is" restaurant. It appears that Spuds-n-More3 is not that much better than Spuds-n-More2 from a customer service rate perspective. Perhaps we will advise Mr. Taylor to request funding from the restaurant's stockholders to implement the Spuds-n-More2 design because the confidence interval suggests that it processes an estimated 14 to 19 more customers per day than does the current restaurant. The extra expense of Spuds-n-More3 above the implementation cost of Spuds-n-More2 for spacing the order and pickup windows farther apart will probably not be offset by the extra income from processing only an estimated 1 to 3 more customers per day.

In the first exercise problem in the next section, we will find out how much of a reduction, if any, we achieved in the half-widths of the confidence intervals by using the CRN technique. Theoretically, the CRN technique produces a positive correlation between the observations in columns B, C, and D of Table L10.1. The positive correlation reduces the value of the sample standard deviation of the observations in the difference columns (columns E, F, and G of Table L10.1). The smaller standard deviation results in a smaller confidence interval half-width (a more precise statement). This is important because, for example, if the third confidence interval was $-10.93 \leq \mu_{(2-3)} \leq -1.79$, which suggests that Spuds-n-More3 processes an estimated 1.79 to 10.93 more customers per day on average than does Spuds-n-More2, then we may not be so quick to rule out Spuds-n-More3 as a serious competitor to Spuds-n-More2 due to the greater range of the confidence interval.